Exercise 31

Find the limit or show that it does not exist.

$$\lim_{x\to\infty} \frac{x^4 - 3x^2 + x}{x^3 - x + 2}$$

Solution

Multiply the numerator and denominator by the reciprocal of the highest power of x in the denominator.

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^4 - 3x^2 + x}{x^3 - x + 2} &= \lim_{x\to\infty} \frac{x^4 - 3x^2 + x}{x^3 - x + 2} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} \\
&= \lim_{x\to\infty} \frac{(x^4 - 3x^2 + x)\frac{1}{x^3}}{(x^3 - x + 2)\frac{1}{x^3}} \\
&= \lim_{x\to\infty} \frac{x - \frac{3}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2} + \frac{2}{x^3}} \\
&= \frac{\lim\limits_{x\to\infty} \left(x - \frac{3}{x} + \frac{1}{x^2}\right)}{\lim\limits_{x\to\infty} \left(1 - \frac{1}{x^2} + \frac{2}{x^3}\right)} \\
&= \frac{\lim\limits_{x\to\infty} x - \lim\limits_{x\to\infty} \frac{3}{x} + \lim\limits_{x\to\infty} \frac{1}{x^2}}{\lim\limits_{x\to\infty} 1 - \lim\limits_{x\to\infty} \frac{1}{x^2} + \lim\limits_{x\to\infty} \frac{2}{x^3}} \\
&= \frac{\infty - 0 + 0}{1 - 0 + 0} \\
&= \infty
\end{aligned}$$